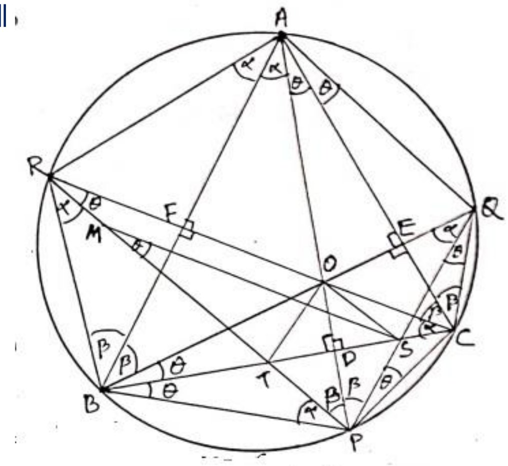
Given :

ΔABC is inscribed in the circle ABC. O is the orthocentre. $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ are altitudes. $\overrightarrow{AD}$,$\overrightarrow{BE}$ and $\overrightarrow{CF}$ cut the circle at P,Q,R respectively. $\overline{PR}$ & $\overline{PQ}$ cuts $\overline{BC}$ at T & S. $\overline{SM}$ $\parallel$



Claim : RM = PT

Construction : Join $\overline{OT}$ & $\overline{OS}$, $\overline{AQ}$, $\overline{QC}$, $\overline{CP}$, $\overline{PB}$, $\overline{BR}$, $\overline{RA}$

Proof:

Let $m\angle CRP = \theta$

$\Longrightarrow m\angle SMP = \theta$ as $\overline{CR} \parallel \overline{SM}$

$m\angle PRC = m\angle PAC = \theta$

Inscribed in the arc PAC

Now in ΔAOE $m\angle AEO = 90$ and

$m\angle OAE = \theta$ $\Longrightarrow$ m$\angle AOE = 90 - \theta$ (Opposite angle)

$\Longrightarrow m\angle BOD = 90 - \theta$

In the ΔBOD m$\angle BDO = 90$ and $m\angle BOD = 90 - \theta \Longrightarrow$ $m\angle BOD = \theta$

m$\angle CBQ =$ m$\angle CAP = \theta \Longrightarrow$PC = CQ

m $\angle CAQ = m\angle CBP = \theta$ $m\angle PQC = m\angle CPQ = \theta$

Let m$\angle BAP = \alpha$ $\Longrightarrow m\angle BRP = m\angle BQP = \alpha$ (inscribed in the arc BAP)

Now in the right ΔAFO, m$\angle AOF = 90 - \alpha$ $\Longrightarrow$ m$\angle COD = 90 - \alpha$

Hence m$\angle OCD = \alpha$ $\Longrightarrow$ m$\angle RCB = \alpha$

m$\angle RCB =$ m$\angle BAP = \alpha \Longrightarrow$ BR = BP

If m$\angle APQ = \beta$, then m$\angle ABQ = \beta$ and $m\angle BOD = 90 - \beta$ as m$\angle BFO$ = 90

Now, m$\angle BOF$= m$\angle COE - 90 - \beta$

$\Longrightarrow$ m$\angle OCE = \beta$ as $m\angle OEC = 90$

$\Longrightarrow$ m$\angle RCA = \beta$ Now m$\angle RCA$ = m$\angle APQ = \beta$ $\Longrightarrow$AR=AQ and

And hence m$\angle ACQ =$ m$\angle APR =$ m$\angle ABR = \beta$

Now in ΔPTD& ΔPSD m$\angle TPD =$ m$\angle SPD = \beta$

m$\angle PDT =$ m$\angle PDS = 90$,$\overline{PD}$ $common$ $\Longrightarrow \Delta PDT \cong \Delta PDS$

$\Longrightarrow$ DT = DS

Similarly in ΔBPD & ΔBOD

$m\angle BDP = m\angle BDO = 90$,$m\angle PBF$= $m\angle OBD = \theta$,$\overline{BD}$ common

$\Longrightarrow \Delta BPD \cong \Delta BOD \Longrightarrow OD = PD$

Now in the quadrilateral OTPS, OD = PD & DT = DS

$\Longrightarrow$ $OTPS$ is a parallelogram

$\Longrightarrow$ $\overline{OT} \parallel \overline{PS}$ and $\overline{OS} \parallel \overline{PT}$ and OS= PT

$\overline{OT} \parallel \overline{PS}$ and $\overline{OS} \parallel \overline{PT}$ and OS=PT

$\Longrightarrow$ $\overline{OS} \parallel \overline{PT} \Longrightarrow \overline{OS} \parallel \overline{RT}$ $\Longrightarrow \overline{OS} \parallel \overline{RM}$

$\overline{OS} \parallel \overline{RM}$ & $\overline{SM} \parallel \overline{CR} \Longrightarrow$ ROSM is a parallelogram

$\Longrightarrow$ OS = RM but OS = PT already proved

Hence RM = PT proved.

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